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Synchronization Stability of Complex Dynamical Networks with Probabilistic Time-Varying Delays

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***Abstract*— A kind of complex dynamical networks with time-varying coupling delays is proposed. By some transformation, the synchronization problem of the complex networks is transferred equally into the stochastic asymptotical stability problem of a group of uncorrelated delay functional differential equations. Different from the common assumptions on the delay in the existing references, the delay in this paper is assumed to be random and its probability distribution is known a prior. In terms of the probability distribution of the delays, a new type of system model with probability-distribution-dependent parameter matrices is proposed, the sufficient condition for delay-dependent asymptotical synchronization stability is derived in the form of linear matrix inequalities, the solvability of derived conditions depends on not only the size of the delay, but also the probability of the delay taking values in some intervals. At last, a numerical example is given to illustrate the feasibility and effectiveness of the proposed method.**

I. INTRODUCTION

Complex network models are often used to describe various interconnected systems of real world, such as the world wide web, food webs, electronic power grids, internet etc [1]–[4]. Since the complexity of real world network, there are various complex network models used to study the dynamics of coupled systems. Synchronization is a basic motion in coupled dynamical networks which has been carefully studied in [5]–[9].

The characteristic of time-delayed coupling is very common in biological and physical systems etc [10]–[12], some of time delays are trivial so can be ignorant, while some others cannot be ignored, such as in long distance communication, traffic congestions etc. Therefore, time delays should be modeled in order to simulate more realistic networks. Wang and Chen [13] analyzed the synchronization based on a simple uniform dynamical network model with the same coupling strength for all connections. Li and Chen [14] extended the model to the one with coupling delays, and derived several synchronization criteria in the form of linear matrix inequality. Gao, lam and chen [5] showed some improved rules to judge the asymptotic stability of complex networks with invariant delays, which are less conservative when compared with Li and Chen's results in [14]. But it is worth noting that most of the existing results on complex networks are concerned with constant delays, little progress has been made towards solving the problem arising from complex networks with time-varying coupling delays. Moreover, in many practical systems, such as networked control systems, the probability distribution of time delay in the interval is an important characteristic for the network conditions [15], [16], the probability of the delay appearing in lower interval is large and long delay happens with a low probability, it may be outside the allowable variation range given in the traditional methods [17]. Therefore, the information of probability distribution of the delay should be employed in the model. Synchronization stability analysis, as one of the fundamental problems for complex networked, still remains unsolved and challenging, which motivates the present study.

In this paper, basing on the above analysis, a class of complex dynamical networks with time-varying coupling delays is proposed. By some transformation, the synchronization problem of the complex networks is transferred equally into the stochastic asymptotical stability problem of a group of uncorrelated delay functional differential equations. It is assumed that the probability of the delays appearing in some intervals can be observed and modeled as a function of the stochastic variable satisfying Bernoulli random binary distribution. In terms of the probability distribution of the delay, a new type of system model with stochastic parameter matrices is proposed, the sufficient condition for delay-dependent asymptotical synchronization stability is derived in the form of linear matrix inequalities. A numerical example is given to illustrate the theoretical results.

II. COMPLEX DYNAMICAL NETWORKS MODEL AND PRELIMINARIES

Consider delayed complex dynamical networks consisting of N identical nodes, in which each node is an m-dimensional dynamical subsystem

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} g_{ij}\Gamma x_j(t-\tau(t)) \qquad (i = 1, 2, \cdots, N) \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots x_{im}(t))^T \in R^m$ is the state vector of the ith node. $f(\cdot) \in R^m$ is a continuously differentiable vector function. The constant $c > 0$ represents

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the coupling strength. $\Gamma = (\gamma_{ij}) \in R^{m\times m}$ is a inner-coupling matrix, if some pairs $(i,j), 1 \le i,j \le m$, with $\gamma_{ij} \neq 0$, then it means two coupled nodes are linked through their ith and jth state variables, $G = (g_{ij})_{N\times N}$ represents the outer-coupling matrix of the networks, in which g_{ij} is defined as follows: if there exist a connection between node ith and node jth $(j \neq i)$, then $g_{ij} = g_{ji} = 1$, otherwise $g_{ij} = g_{ji} = 0, (j \neq i)$, and the diagonal elements of matrix G are defined by

$$g_{ii} = -\sum_{j=1,j\neq i}^{N} g_{ij} = -\sum_{j=1,j\neq i}^{N} g_{ji} \quad (i = 1,2,\cdots N) \tag{2}$$

$\tau(t)$ is a bounded time-varying delay.

Assumption 1: There exist three constants τ_1, τ_2 and $\beta_0 \in [0,1]$, where $0 \le \tau_1 \le \tau_2$, such that

(1) $0 \le \tau(t) \le \tau_2$

(2) $the\ probability\ of\ \tau(t)\ taking\ \ values\ in\ [0,\tau_1)\ is\ \beta_0\ and\ in\ [\tau_1,\tau_2]\ is\ 1-\beta_0$

Define two sets

$$\begin{aligned}\Omega_1 &= \{t:\ \tau(t) \in [0,\ \tau_1)\}\\ \Omega_2 &= \{t:\ \tau(t) \in [\tau_1,\ \tau_2]\}\end{aligned}$$

Obviously $\Omega_1 \cup \Omega_2 = R^+$ and $\Omega_1 \bigcap \Omega_2 = \phi\ (empty)$. From the definitions of Ω_1 and Ω_2, it can be seen that $t \in \Omega_1$ means the event $\tau(t) \in [0,\tau_1)$ occurs and $t \in \Omega_2$ means the event $\tau(t) \in [\tau_1,\tau_2]$ occurs. Based on the above two sets, the following two functions are defined

$$\tau_1(t) = \begin{cases}\tau(t) & for\ \ t \in \Omega_1\\ 0 & for\ \ t \in \Omega_2\end{cases}$$

$$\tau_2(t) = \begin{cases}\tau(t) & for\ \ t \in \Omega_2\\ \tau_1 & for\ \ t \in \Omega_1\end{cases}$$

Furthermore, we can define a stochastic variable $\beta(t)$ as

$$\beta(t) = \begin{cases}1 & for\ \ t \in \Omega_1\\ 0 & for\ \ t \in \Omega_2\end{cases}$$

Assumption 2: $\beta(t)$ is a Bernoulli distributed sequence with

$$\begin{aligned}P\{\beta(t) = 1\} &= E\{\beta(t)\} = \beta_0\\ P\{\beta(t) = 0\} &= 1 - E\{\beta(t)\} = 1 - \beta_0\end{aligned}$$

Remark 1: The introduction of $\beta(t)$ is motivated by [17], where the Bernoulli distributed sequence $\beta(t)$ is used to model the missing message of the system. Different from [17], $\beta(t)$ is used in this paper to describe the probability of the random delays appearing in different intervals.

Definition 1: The delayed dynamical networks (1) is said to achieve asymptotic synchronization if

$$x_1(t) = x_2(t) = \cdots = x_N(t) = s(t) \quad as \quad t \to \infty \tag{3}$$

where $s(t)$ is a solution of an isolate node and satisfies $\dot{s}(t) = f(s(t))$.

To obtain the main results, the following lemmas are needed.

Lemma 1: Consider the delayed dynamical networks (1), the eigenvalues of outer coupling matrix G are denoted by

$$0 = \lambda_1 > \lambda_2 \ge \lambda_3 \ge \cdots \ge \lambda_N$$

if the following $N-1$ of m-dimensional time-varying delayed differential equations are stochastic asymptotically stable about their zero solution

$$A\xi(t) + (\beta(t) - \beta_0)B\xi(t) = 0 \tag{4}$$

where

$$\begin{aligned}A &= \begin{bmatrix} J(t) & \beta_0 c\lambda_k\Gamma & 0 & (1-\beta_0)c\lambda_k\Gamma & 0 & -I\end{bmatrix}\\ B &= \begin{bmatrix} 0 & c\lambda_k\Gamma & 0 & -c\lambda_k\Gamma & 0 & 0\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\xi^T(t) = [\eta_k^T(t) \quad & \eta_k^T(t-\tau_1(t)) \quad \eta_k^T(t-\tau_1)\\ & \eta_k^T(t-\tau_2(t)) \quad \eta_k^T(t-\tau_2) \quad \dot{\eta}_k^T(t)]\end{aligned}$$

$J(t)$ is the Jacobian of $f(x(t))$ at $s(t)$, then the synchronized states (3) are asymptotically stable.

Proof: Define error vectors as

$$e_i(t) = x_i(t) - s(t) \quad (i = 1,2,\cdots N) \tag{5}$$

According to the networks (1), the error system is described by

$$\dot{e}_i(t) = f(x_i(t)) - f(s(t)) + c\sum_{j=1}^{N} g_{ij}\Gamma e_j(t-\tau(t)) \tag{6}$$

Since $f(\cdot)$ is continuous differentiable, linearizing the controlled network (1) on the homogenous stationary state $s(t)$ leads to

$$\dot{e}(t) = e(t)J^T(t) + Ge(t-\tau(t))\Gamma^T \tag{7}$$

where $e(t) = (e_1(t), e_2(t), \cdots e_N(t))^T \in R^{Nm}$, there exist an orthogonal matrix $\phi = (\phi_1, \phi_2, \cdots \phi_N) \in R^{N\times N}$, such that

$$G^T\phi_k = \lambda_k\phi_k \quad (k = 1,2,\cdots,N) \tag{8}$$

Using the nonsingular transform $e(t) = \phi\eta(t)$, then (7) can be expanded into the following equations

$$\dot{\eta}(t) = \eta(t)J^T(t) + c\Lambda\eta(t-\tau(t))\Gamma^T \tag{9}$$

where $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_N\}$, furthermore, we can obtain

$$\dot{\eta}_k(t) = J(t)\eta_k(t) + \lambda_k\Gamma\eta_k(t-\tau(t)) \quad (k = 1,2,\cdots,N) \tag{10}$$

where η_k^T is the kth row of $\eta(t)$. Note that $\lambda_1 = 0$ corresponding to the synchronization of the system states (3), where the state $s(t)$ is an orbitally solution of the isolate nodes. If the following $N-1$ pieces of m- dimensional linear time-varying delayed differential equations

$$\dot{\eta}_k(t) = J(t)\eta_k(t) + \lambda_k\Gamma\eta_k(t-\tau(t)) \quad (k = 2,3,\cdots,N) \tag{11}$$

are asymptotically stable, then $e(t)$ will tend to the origin asymptotically, which implies that the synchronized states (3) are asymptotically stable.

By using the new function $\tau_i(t)\ \ (i=1,2)$ and $\beta(t)$, the system (11) can be rewritten as

$$\begin{aligned}\dot{\eta}_k(t) &= J(t)\eta_k(t)+\beta(t)c\lambda_k\Gamma\eta_k(t-\tau_1(t))\\ &\quad +(1-\beta(t))c\lambda_k\Gamma\eta_k(t-\tau_2(t))\end{aligned} \tag{12}$$

which can be further expressed as

$$A\xi(t)+(\beta(t)-\beta_0)B\xi(t)=0 \tag{13}$$

Hence the stability problem of $N\times m$-dimensional system (1) is converted into the stability problem of $N-1$ independent of m-dimensional linear stochastic system (4). ■

Since (4) is a stochastic system, for the stability analysis of (4), the following definition is needed.

Definition 2: For a given function $V : C^b_{F_0}([-\tau_2,0],R^n)\times\mathbb{S}$, its infinitesimal operator $\mathcal{L}[\cdot]$, is defined as

$$\mathcal{L}V(x_t)=lim_{\Delta\to 0^+}\frac{1}{\Delta}[E(V(x_{t+\Delta}|x_t))-V(x_t)] \tag{14}$$

Moreover, we also need the following lemmas.

Lemma 2: $Q_{1i},Q_{2i}\ (i=1,2)$ and Q are constant matrices of appropriate dimensions, $\tau_i(t),\ (i=1,2)$ is function of t and satisfies $0\le\tau_1(t)\le\tau_0\le\tau_2(t)\le\tau_M$, then

$$\begin{aligned}&Q+[\tau_1(t)Q_{11}+(\tau_0-\tau_1(t))Q_{21}]\\ &+[(\tau_2(t)-\tau_0)Q_{12}+(\tau_M-\tau_2(t))Q_{22}]<0\end{aligned} \tag{15}$$

if and only if

$$\tau_0Q_{11}+(\tau_M-\tau_0)Q_{12}+Q < 0 \tag{16}$$
$$\tau_0Q_{11}+(\tau_M-\tau_0)Q_{22}+Q < 0 \tag{17}$$
$$\tau_0Q_{21}+(\tau_M-\tau_0)Q_{12}+Q < 0 \tag{18}$$
$$\tau_0Q_{21}+(\tau_M-\tau_0)Q_{22}+Q < 0 \tag{19}$$

III. SYNCHRONIZATION STABILITY CRITERIA IN COMPLEX DYNAMICAL NETWORKS

Clearly, synchronization of dynamical networks (1) is equivalent to the stochastic stability of dynamical networks (4) about zero solution. A sufficient condition for delay-dependent stochastic asymptotical stability of system (4) is given as follows.

Theorem 1: For given scalars $0\le\tau_1\le\tau_2$, the system (4) is asymptotically stable, if there exist matrices $P>0,Q_i>0,(i=1,2),R_i>0,(i=1,2,3)$ and $N_i,M_i,V_i,T_i(i=1,2,3,4,5,6)$ and $S_i,(i=1,2)$ of appropriate dimensions such that the following matrix inequality holds

$$\begin{bmatrix}\Sigma_{11} & \Sigma_{12}^i & \Sigma_{13}\\ (\Sigma_{12}^i)^T & \Sigma_{22} & 0\\ (\Sigma_{13})^T & 0 & \Sigma_{33}\end{bmatrix}<0,(i=1,\cdots,4) \tag{20}$$

where

$$\Sigma_{11}=\begin{bmatrix}\phi_{11}&\phi_{12}&\phi_{13}&\phi_{14}&\phi_{15}&\phi_{16}\\ \phi_{12}^T&\phi_{22}&\phi_{23}&\phi_{24}&\phi_{25}&\phi_{26}\\ \phi_{13}^T&\phi_{23}^T&\phi_{33}&\phi_{34}&\phi_{35}&\phi_{36}\\ \phi_{14}^T&\phi_{24}^T&\phi_{34}^T&\phi_{44}&\phi_{45}&\phi_{46}\\ \phi_{15}^T&\phi_{25}^T&\phi_{35}^T&\phi_{45}^T&\phi_{55}&\phi_{56}\\ \phi_{16}^T&\phi_{26}^T&\phi_{36}^T&\phi_{46}^T&\phi_{56}^T&\phi_{66}\end{bmatrix}$$

$$\begin{aligned}
\Sigma_{12}^1 &= \begin{bmatrix}\sqrt{\tau_1}N & \sqrt{\tau_2-\tau_1}V\end{bmatrix}\\
\Sigma_{12}^2 &= \begin{bmatrix}\sqrt{\tau_1}N & \sqrt{\tau_2-\tau_1}T\end{bmatrix}\\
\Sigma_{12}^3 &= \begin{bmatrix}\sqrt{\tau_1}M & \sqrt{\tau_2-\tau_1}V\end{bmatrix}\\
\Sigma_{12}^4 &= \begin{bmatrix}\sqrt{\tau_1}M & \sqrt{\tau_2-\tau_1}T\end{bmatrix}
\end{aligned}$$

$$\Sigma_{13}=\begin{bmatrix}S_1 & 0\\ 0 & \sqrt{\beta_0(1-\beta_0)}c\lambda_k\Gamma\\ 0 & 0\\ 0 & \sqrt{\beta_0(1-\beta_0)}c\lambda_k\Gamma\\ 0 & 0\\ S_2 & 0\end{bmatrix}$$

$$\Sigma_{22}=\begin{bmatrix}-R_1 & 0\\ 0 & -R_2\end{bmatrix}$$

$$\Sigma_{33}=\begin{bmatrix}-R_3 & 0\\ 0 & -R_3\end{bmatrix}$$

$$\begin{aligned}
\phi_{11} &= Q_1+Q_2+N_1+N_1^T+S_1J(t)+J^T(t)S_1^T\\
\phi_{12} &= -N_1+M_1+N_2^T+\beta_0c\lambda_kS_1\Gamma\\
\phi_{13} &= -M_1+V_1+N_3^T\\
\phi_{14} &= -V_1+T_1+N_4^T+(1-\beta_0)c\lambda_kS_1\Gamma\\
\phi_{15} &= -T_1+N_5^T\\
\phi_{16} &= P-S_1+N_6^T+J^T(t)S_2^T\\
\phi_{22} &= M_2+M_2^T-N_2-N_2^T\\
\phi_{23} &= -M_2+V_2-N_3^T+M_3^T\\
\phi_{24} &= M_4^T-N_4^T-V_2+T_2\\
\phi_{25} &= M_5^T-N_5^T-T_2\\
\phi_{26} &= M_6^T-N_6^T+\beta_0c\lambda_k\Gamma^TS_2^T\\
\phi_{33} &= -Q_1-M_3-M_3^T+V_3+V_3^T\\
\phi_{34} &= -M_4^T+T_3-V_3+V_4^T\\
\phi_{35} &= -M_5^T-T_3+V_5^T\\
\phi_{36} &= -M_6^T+V_6^T\\
\phi_{44} &= -V_4-V_4^T+T_4+T_4^T\\
\phi_{45} &= -T_4-V_5^T+T_5^T\\
\phi_{46} &= -V_6^T+T_6^T+(1-\beta_0)c\lambda_k\Gamma^TS_2^T\\
\phi_{55} &= -Q_2-T_5-T_5^T\\
\phi_{56} &= -T_6^T\\
\phi_{66} &= -S_2-S_2^T+\tau_1R_1+(\tau_2-\tau_1)R_2\\
N^T &= \begin{bmatrix}N_1^T & N_2^T & N_3^T & N_4^T & N_5^T & N_6^T\end{bmatrix}\\
M^T &= \begin{bmatrix}M_1^T & M_2^T & M_3^T & M_4^T & M_5^T & M_6^T\end{bmatrix}\\
V^T &= \begin{bmatrix}V_1^T & V_2^T & V_3^T & V_4^T & V_5^T & V_6^T\end{bmatrix}\\
T^T &= \begin{bmatrix}T_1^T & T_2^T & T_3^T & T_4^T & T_5^T & T_6^T\end{bmatrix}
\end{aligned}$$

Proof: Construct a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V(\eta_{kt}) &= \eta_k^T(t)P\eta_k(t) + \int_{t-\tau_1}^{t} \eta_k^T(s)Q_1\eta_k(s)ds \\ &+ \int_{t-\tau_2}^{t} \eta_k^T(s)Q_2\eta_k(s)ds \\ &+ \int_{t-\tau_1}^{t}\int_{s}^{t} \dot{\eta}_k^T(v)R_1\dot{\eta}_k(v)dvds \\ &+ \int_{t-\tau_2}^{t-\tau_1}\int_{s}^{t} \dot{\eta}_k^T(v)R_2\dot{\eta}_k(v)dvds \end{aligned} \tag{21}$$

where $P, Q_i, R_i > 0, \quad (i = 1, 2)$

Using the infinitesimal operator (14) and the derivative leads to the following equality

$$\begin{aligned} \mathcal{L}V(\eta_{kt}) &= 2\dot{\eta}_k^T(t)P\eta_k(t) + \eta_k^T(t)(Q_1+Q_2)\eta_k(t) \\ &-\eta_k^T(t-\tau_1)Q_1\eta_k(t-\tau_1) \\ &-\eta_k^T(t-\tau_2)Q_2\eta_k(t-\tau_2) \\ &+\dot{\eta}_k^T(t)(\tau_1R_1+(\tau_2-\tau_1)R_2)\dot{\eta}_k(t) \\ &-\int_{t-\tau_1}^{t} \dot{\eta}_k^T(s)R_2\dot{\eta}_k(s)ds \\ &-\int_{t-\tau_2}^{t-\tau_1} \dot{\eta}_k^T(s)R_2\dot{\eta}_k(s)ds \end{aligned} \tag{22}$$

Employing the free matrix method, we have

$$2\xi^T(t)N[\eta_k(t)-\eta_k(t-\tau_1(t)) - \int_{t-\tau_1(t)}^{t} \dot{\eta}_k(s)ds] = 0 \tag{23}$$

$$2\xi^T(t)M[\eta_k(t-\tau_1(t))-\eta_k(t-\tau_1) - \int_{t-\tau_1}^{t-\tau_1(t)} \dot{\eta}_k(s)ds] = 0 \tag{24}$$

$$2\xi^T(t)V[\eta_k(t-\tau_1)-\eta_k(t-\tau_2(t)) - \int_{t-\tau_2(t)}^{t-\tau_1} \dot{\eta}_k(s)ds] = 0 \tag{25}$$

$$2\xi^T(t)T[\eta_k(t-\tau_2(t))-\eta_k(t-\tau_2) - \int_{t-\tau_2}^{t-\tau_2(t)} \dot{\eta}_k(s)ds] = 0 \tag{26}$$

$$2\xi^T(t)S_{12}[A\xi(t)+(\beta(t)-\beta_0)B\xi(t)] = 0 \tag{27}$$

where

$$S_{12}^T = \begin{bmatrix} S_1^T & 0 & 0 & 0 & 0 & S_2^T \end{bmatrix}$$

There exist $R_i > 0, \quad (i = 1, 2, 3)$, such that

$$\begin{aligned} &-2\xi^T(t)N\int_{t-\tau_1(t)}^{t} \dot{\eta}_k(s)ds \\ \le\ & \tau_1(t)\xi^T(t)NR_1^{-1}N^T\xi(t) \\ &+\int_{t-\tau_1(t)}^{t} \dot{\eta}_k^T(v)R_1\dot{\eta}_k(s)ds \end{aligned} \tag{28}$$

$$\begin{aligned} &-2\xi^T(t)M\int_{t-\tau_1}^{t-\tau_1(t)} \dot{\eta}_k(s)ds \\ \le\ & (\tau_1-\tau_1(t))\xi^T(t)MR_1^{-1}M^T\xi(t) \\ &+\int_{t-\tau_1}^{t-\tau_1(t)} \dot{\eta}_k^T(s)R_1\dot{\eta}_k(s)ds \end{aligned} \tag{29}$$

$$\begin{aligned} &-2\xi^T(t)V\int_{t-\tau_2(t)}^{t-\tau_1} \dot{\eta}_k(s)ds \\ \le\ & (\tau_2(t)-\tau_1)\xi^T(t)VR_2^{-1}V^T\xi(t) \\ &+\int_{t-\tau_2(t)}^{t-\tau_1} \dot{\eta}_k^T(s)R_2\dot{\eta}_k(s)ds \end{aligned} \tag{30}$$

$$\begin{aligned} &-2\xi^T(t)T\int_{t-\tau_2}^{t-\tau_2(t)} \dot{\eta}_k(s)ds \\ \le\ & (\tau_2-\tau_2(t))\xi^T(t)TR_2^{-1}T^T\xi(t) \\ &+\int_{t-\tau_2}^{t-\tau_2(t)} \dot{\eta}_k^T(v)R_2\dot{\eta}_k(s)ds \end{aligned} \tag{31}$$

$$\begin{aligned} &2\xi^T(t)S_{12}(\beta(t)-\beta_0)B\xi(t) \\ \le\ & \xi^T(t)(S_{12}R_3^{-1}S_{12}^T \\ &+(\beta(t)-\beta_0)^2B^TR_3B)\xi(t) \end{aligned} \tag{32}$$

Adding (23) – (27) to the right of (22) and Substituting (28) – (32) into (22) and taking expectation on the both sides of (22), we can obtain

$$\begin{aligned} E\{\mathcal{L}V(\eta_{kt})\} \le\ & \xi^T(t)[\Sigma_{11} + S_{12}R_3^{-1}S_{12}^T \\ &+\beta_0(1-\beta_0)B^TR_3B + \tau_1(t)NR_1^{-1}N^T \\ &+(\tau_1-\tau_1(t))MR_1^{-1}M^T \\ &+(\tau_2(t)-\tau_1)VR_2^{-1}V^T \\ &+(\tau_2-\tau_2(t))TR_2^{-1}T^T]\xi(t) \end{aligned} \tag{33}$$

By schur complement from (20) and Lemma 2, we can conclude

$$\begin{aligned} &\Sigma_{11} + S_{12}R_3^{-1}S_{12}^T + \beta_0(1-\beta_0)B^TR_3B \\ &+\tau_1(t)NR_1^{-1}N^T + (\tau_1-\tau_1(t))MR_1^{-1}M^T \\ &+(\tau_2(t)-\tau_1)VR_2^{-1}V^T + (\tau_2-\tau_2(t))TR_2^{-1}T^T < 0 \end{aligned}$$

It's easy to see $E\{\mathcal{L}V(\eta_{kt}\} < 0$, then by the Lyapunov stability theory, we know that the system (4) are stochastic asymptotically stable, according to lemma 1, the asymptotic synchronization defined in (3) is achieved, the proof is completed. ■

IV. NUMERICAL EXAMPLES

Example 1: Consider a 10-node network, in which each node is a simple three-dimensional linear system describe in

Refs [18]

$$\begin{cases} \dot{x}_{i1}(t) = -x_{i1}(t) \\ \dot{x}_{i2}(t) = -2x_{i2}(t) \\ \dot{x}_{i3}(t) = -3x_{i3}(t) \end{cases} \tag{34}$$

which is asymptotically stable at the equilibrium point $s(t) = (0,0,0)^T$, and its Jacobin matrix is $J(t) = diag\{-1,-2,-3\}$, for simplicity. We suppose that the inner-coupling matrix is $\Gamma = diag(1,1,1)$, and the coupling matrix G is

$$\begin{bmatrix} -4 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -3 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & -2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -2 \end{bmatrix}$$

obviously, G is an irreducible symmetric matrix. Therefore, if the delayed equations (4) are asymptotically stable, then the synchronized state $s(t)$ is asymptotically stable. some work has been done to complex networks with constant time delays[7][13], however, here we consider the complex networks with time varying delays, which are more common in practical. When the delay is random and its probability distribution is known a prior, using Theorem 1, we can obtain Table I and Table II, which list the maximum allowable bounds for different c, β_0, and τ_1.

TABLE I
MAXIMUM ALLOWABLE τ_2 FOR DIFFERENT β_0, $(c = 0.1)$

τ_1	0.1	0.3	0.5
$\beta_0 = 0$	3.2210	3.2494	3.3274
$\beta_0 = 0.1$	3.5802	3.5798	3.6385
$\beta_0 = 0.7$	10.6175	10.0946	9.7752
$\beta_0 = 0.9$	31.6453	29.5739	27.9635

TABLE II
MAXIMUM ALLOWABLE τ_2 FOR DIFFERENT β_0, $(c = 0.5)$

τ_1	0.1	0.3	0.5
$\beta_0 = 0$	0.5291	0.5306	0.5553
$\beta_0 = 0.1$	0.6104	0.5678	0.5619
$\beta_0 = 0.7$	1.7763	1.1506	0.6442
$\beta_0 = 0.9$	5.1324	2.8006	0.8459

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In this paper, the synchronization problem has been investigated for complex dynamical networks with probabilistic interval time-varying delays. Based on the information of probability distribution of time delay, a new model of the system, which has stochastic parameter matrices, has been proposed, Basing on the new model, the sufficient condition for delay-dependent asymptotical synchronization stability is derived in the form of linear matrix inequalities. Illustrative examples are presented, which show the efficiency of the derived results.

B. Future Works

It should be pointed out that the method in the present paper can also be extended to the case when the probability of the delay taking values in series of intervals can be observed, this work will be left for our future research.

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